IMPORTANT QUESTIONS FOR BOARD EXAM

CLASS – XII

MATHEMATICS

1. Evaluate : $\int \frac{x^2}{x^4+x^2-2}dx$
2. Find the area enclosed by the parabola $y^2 = x$ and the line y + x = 2.
3. Using integration find the area of the region bounded by the triangle whose vertices are (- 1, 2), (1, 5) and (3 , 4).
4. Find the area of the region above the x – axis, included between the curves $y^2 = ax$ and $x^2 + y^2 = 2ax$.
5. Using the integration, find the area bounded by the curve |x| + |y| = 1.
6. Form the differential equation of the family of circles in the first quadrant which touches the coordinate axes.
7. Form the differential equation of the family of ellipses having their foci on x-axis and centre at the origin.
8. Find the particular solution of the differential equation (x – y) (dx + dy) = dx – dy, given that y = -1 when x = 0.
9. Solve the differential equation ($1 + x^2$) dy + 2xy dx = cot x dx
10. Solve the differential equation ($x^2 + 3xy + y^2$) dx - x^2 dy = 0.
11. If $\vec{a}, \vec{b}, and\ \vec{c}\ are$ mutually perpendicular unit vectors, then find the value of | $2\vec{a} + \vec{b} + \vec{c}$ |.
12. Find vectors of magnitudes 10√3 units that are perpendicular to the plane of vectors i+ 2j +k and –i+ 3j + 4k.
13. Find the coordinates of the foot of perpendicular drawn from the point A(1, 8, 4) to the line joining the points B(0, -1, 3) and C(2, -3, -1).
14. Find the vector and cartisian equations of the line through the points (1, -1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, -1,0) and (1, 2, -1), (2, 1, 1).
15. Find the distance of the point (1, -2, 3) from the plane x – y + z measured along a line parallel to the line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{-6}$.
16. Find the image of the point having position vector i+3 j + 4k in the plane $\vec{r}$. (2i – j +k) + 3 = 0.
17. Show that the lines $\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z+3}{-5}\ and\ \frac{x-8}{7} = \frac{2y-8}{2} = \frac{z-5}{3}$ are coplanar.
18. Find the equation of the plane passing through the line of intersection of the planes x + 2y + 3z – 5 = 0 and 3x – 2y – z + 1 =0 and cutting off equal intercepts on x – axis and z axis.
19. Find the equation of the plane passing through the line of intersection of the planes x – 2y + z =1 and 2x + y + z = 8 and parallel to the line with direction ratios 1,2,1. Also find the perpendicular distance of (1, 3, 2) from the obtained plane.
20. A manufacturing company makes two types of teaching aids A an B of mathematics for class 12. Each type of A requires 9 labour hours for fabricating and 1 hour for finishing. Each type of B requires 12 hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week ar 180 and 30 respectively. The company makes a profit of Rs 80 on each piece of type aA and Rs 120 on each piece of type B. how many pieces of

type A and type B should be made per week to get maximum profit? Solve it graphically to find maximum profit per week.

21. If a young man rides his motor cycle at 25 km per hour, he has to spend Rs 2 per km on petrol; if he rides at a faster speed of 40 km per hour, the petrol cost incteases to Rs 5 per km. he has Rs 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it.
22. Two tailors A and B are paid Rs 225 and Rs 300 per day respectively. A can stich 9 shirts and 6 pants while B can stich 15 shirts and 6 pants per day. Form a linear programming problem to minimize the labour cost to produce atleat 90 shirts and 48 pants and solve it graphically.
23. A catering agency has two kitchens to prepare food at two places A and B. from these places mid day meal is to be supplied to three different schools situated at P, Q and R. the monthly requirements of the schools are respectively 40, 40 and 50 food packets. A packet contains lunch for 100 students. Preparing capacity of kitchens A and B are 60 and 70 packets per month respectively. The transportation cost per packet from the kitchens to schools is given below:

Transportation cost per packet (in Rs)		
To	From	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to school so that the cost of transportation is minimum? Also find the minimum cost.

24. A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of the cases are they likely to contradicting each other in stating the same fact? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A?
25. An article manufactured be a company consists of two parts A and B. in the process of manufacture of part A, 9 out of 104 parts may be defective. Similarly, 5 out of 100 parts are likely to be defective in the manufacture of part B. calculate the probability that the article manufactured will not be defective.
26. A and B throw a pair of dice tun by turn. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of a getting the prize is 9/17.
27. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both clubs. Find the probability of the lost card being a club.
28. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30% and the prescription of a certain drug reduces its chance by 25%. At a time a patient can choose any one of two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.

29. A doctor is to visit a patient. From the past experience, it is known that the probability that he will come by public transport, scooter, taxi or personal car are respectively 1/10, 1/5, 3/10, 2/5. The probabilities that he will be late are ¼, 1/3, 1/12 if he comes by public transport, scooter of taxi respectively, but if he comes by personal car he will not be late. When he arrives, he is late. What is the probability that he came by scooter? Do you think he should use public transport and why?
30. 2 bad eggs are accidentally mixed with 10 good ones. If three eggs are drawn at random, find the probability distribution of bad eggs drawn. Also find the mean and variance of the distribution.
31. The probability of a shooter hitting a target is ¾. How maximum number of times must he fire so that the probability of hitting the target atleast once is more than 99%?
32. Prove that the function f: N → N defined by $f(x) = x^2 + x + 1$ is one one but not onto.
33. If f : R → R is a function defined by f(x) = ax + b for all x ϵ R, then find the constants a and b such that fof = I_R.
34. Let A = R – {3} and B = R – {1}. Consider the function f : A → B defined by f(x) = $\frac{x-2}{x-3}$. show that f is one one and onto. Hence find f^{-1}.
35. If the function f : R → R be defined by f(x) = 2x – 3 and g: R → R by $g(x) = x^3 + 5$, then find the value of $(fog)^{-1}(x)$.
36. Let A = N × N and * be a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). show that * is commutative and associative. Find the identity element for * on A, if any. Also write the inverse element of the element (3, -5) in A.
37. Define a binary operation * on the set {0, 1,2, 3, 4, 5} as a*b = (a + b) (mod 6). Show that zero is the identity for this operation and each element a except 0 of the set is invertible with 6 – a being the inverse of a. also write the operation table for the given operation.
38. Solve the equation $\cos^{-1}(\frac{x^2-1}{x^2+1}) + \tan^{-1}(\frac{2x}{x^2-1}) = 2\pi/3$.
39. A trust caring for handicapped children gets Rs 30000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them, and deposited the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly. What percent of interest shout the trust get from the bank so to get a total of Rs 1800 every month? Use matrix method, to find the rate of interest. Do you think people should donate to such trusts?
40. If A, B are dkew symmetric matrices and AB = BA, then show that AB is symmetric.
41. Find the inverse of the following matrices using elementary transformation:

(i) $\begin{matrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{matrix}$

(ii) $\begin{matrix} -1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & -1 & 1 \end{matrix}$

42. Using the properties of determinants prove that $\begin{matrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{matrix}$ = $(1+a^2+b^2)^3$.

43. Prove that $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (c+a)^2 & bc \\ ca & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

44. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, prove that $A^2 - 4A - 5I = 0$. Hence find A^{-1}.

45. Solve the following system of equations, using matrix method: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$; x,y,z≠0.

46. 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and the third group contains vigilant and obedient students. Double the number in the second group gives 13, while the combined strength of first and second group is four times that of the third group. Using matrix method, find the number of students in each group. Apart from the values, hard work, honesty and respect for law, vigilance and obedience, suggest one more value, which in your opinion, the school should consider for awards.

47. If $A = \begin{bmatrix} 3 & -2 & 1 \\ 2 & 1 & -3 \\ -1 & 2 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the following system of equations: $3x - 2y + z = 2$, $2x + y - 3z = -5$, $-x + 2y + z = 6$.

48. The sum of three numbers is 20. If we multiply the first number by 2 and add the second number to the result and subtract the third number, we get 23. By adding second and third numbers to three times the first number, we get 46. Represent the above problem algebraically and use matrix method to solve the problem.

49. Discuss the continuity of the following function at x = 0: $f(x) = (x^4 + 2x^3 + x^2)/\tan^{-1}x$, x≠0 and f(x) = 0 at x = 0.

50. If the function $f(x) = (1 - \sin^3 x)/3\cos^2 x$, $x<\pi/2$, $f(x) = a$ if $x = \pi/2$ and $f(x) = b(1 - \sin x)/(\pi - 2x)^2$ if $x>\pi/2$ is continuous at $x = \pi/2$, find the values of a and b.

51. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then prove that $\frac{dy}{dx} = -\frac{1}{(1+x)}2$

52. If $\frac{x}{x-y} = log\frac{a}{x-y}$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$

53. Differentiate $(\sin x)^x + (\cos x)^{\tan x}$ with respect to x.

54. If x = a(θ - sin θ), y = a (1 – cosθ), find y'' at θ = π/2.

55. If y = tan x + sec x, prove that $y'' = \frac{\cos x}{(1-\sin\ )^2}$

56. Water is leaking from a conical funnel at the rate of 5 cubic cm/sec. if the radius of the base of the funnel is 10 cm and its height is 20 cm, find the rate at which the water level is dropping when it is 5 cm from the top.

57. If the tangent to the curve $y = x^3 + ax + b$ at P(1, -6) is parallel to the line y – x = 5, find the values of a and b.

58. Find the equation of tangent and normal to the curve $x = a\sin^3 t$ and $y = a\cos^3 t$ at t=π/4.

59. Show that the normal at any point θ to the curve x = a cos θ + a θ sin θ, y = a sin θ – aθ cosθ is at a constant distance from the origin.

60. Find the condition that the curves $2x = y^2$ and 2 x y = k intersect orthogonally.

61. Find the intervals in which the function f(X) = log x(1+ x) – x/(1+x) is strictly increasing or decreasing.

62. Find the values of x for which $f(x) = [x(x-2)]^2$ is an increasing function. Also find the points on the curve, where the tangent is parallel to x – axis.
63. Find the stationary points of the function $f(x) = 3x^4 - 8x^3 + 6x^2$ and distinguish between them. Also find the local maximum and local minimum values if they exists.
64. Find the point on the curve $y = x/(1 + x^2)$, where the tangent to the curve has the greatest slope.
65. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$.
66. Show that the semi vertical angle of the cone of the maximum volume and of given slant height is $\cos^{-1}(1/\sqrt{3})$.
67. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle so that it may produce the largest area of the window.
68. Find the coordinates of the point on the parabola $y = x^2 + 7x + 2$ which is closed to the straight line $y = 3x - 3$. Also find the shortest distance of the point.
69. Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$. Also find the maximum volume.
70. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given right circular cone is half that of the cone.